Performance Analysis of Group-Blind Multiuser Detectors for Synchronous CDMA

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***Abstract* — Blind multiuser detectors are attractive for the suppression of interference in a CDMA environment. This paper deals with the performance of group blind multiuser detector for synchronous CDMA is analyzed. The blind multi user detectors are Direct Matrix Inversion(DMI),Subspace and group blind multiuser detector. The performance analysis is performed by means of the Signal to Interference Noise Ratio(SINR) and Bit Error Rate(BER). The numerical results are plotted as variation of SINR Vs SNR, K and M, SINR with respect to correlation coefficient(ρ) and BER Vs Number of samples(M) for three detectors using MATLAB software. The gain rises in group blind multiuser detector over the DMI and subspace detectors. The comparison is carried out for equicorrelated signals for mathematical simplicity.**

***Keywords* - Blind multiuser detector, CDMA, group blind multiuser detector, signal subspace.**

I. Introduction

CDMA today is a widely preferred as multiple access technique. Considerable research is done multiuser detection [1] with a special emphasis on adaptive multiuser detection techniques [2] which substantially increase the capacity of CDMA systems. In particular blind multiuser detection techniques allow for a multiuser detector to work with prior knowledge of only the desired user's signature sequence. These have been implemented mainly using two approaches direct matrix inversion (DMI) [3], [1] and subspace projection of signal [4], [5], [6]. They are useful in suppressing interference in CDMA downlinks, where the mobile receiver knows only its spreading sequence. In CDMA uplinks, typically a base station receiver has the knowledge of the spreading sequences of a group of users (users within its cell).This additional users spreading sequences can be used to achieve performance gains [2]. Such idea was first made use in [7]-[9], where many linear and nonlinear detectors were developed for synchronous CDMA systems, which improved the subspace blind method in [4], [5] by taking into account the knowledge of the spreading sequences of other users in cell. In this paper a more generalized version of idea in [7] and [9] is considered to develop linear group blind multiuser detection techniques that suppress intracell interference, using the knowledge of the spreading sequences and the estimated multipath channels of a group of known users, while suppressing the intercell interference blindly. It is shown through simulations that such group blind multiuser techniques outperform the blind methods in CMDA uplink environment.

In this paper we see the CDMA signal model used in section II, the blind multiuser detectors and the group blind multiuser detector are given in section III, section IV gives the SINR calculation for the detectors, in section V simulation results and section VI conclusions are made based on simulations.

II. Syncronous CDMA Signal Model

A baseband K-user, time-invariant synchronous Additive White Gaussian Noise (AWGN) system, employing periodic (short) spreading sequences, and operating with a coherent BPSK modulation format. The continuous time waveform received by a given user in such a system can be modelled as follows [4].

$$r(t) = \sum_{k=1}^{K} A_k \sum_{i=0}^{M-1} b_k[i] s_k(t-iT) + n(t), 0 \le t \le MT \quad (1)$$

where M is the number of data symbols per user in the data frame of interest, T is the symbol interval; A_k, $\{b_k[i]\}_{i=0}^{M-1}$ and $S_K(t)$denote respectively the received complex amplitude, the transmitted symbol stream and the normalized signalling waveform of the K^{th} user and *n(t)* is the baseband complex Gaussian ambient noise with independent real and imaginary components and with power spectral density σ^2 . It is assumed that for each user , $\{b_k[i]\}_{i=0}^{M-1}$ is a collection of independent equiprobable ±1 random variables, and the symbol streams of different users are independent. For direct sequence spread-spectrum format, each user's signalling waveform is of the form

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} c_{j,k} \psi(t - jT_c), \quad 0 \le t < T \quad (2)$$

where is the processing gain; $\{c_{j,k}\}_{j=0}^{N-1}$ is a signature sequence of ±1's assigned to the kth user; and $\psi(\cdot)$is a chip waveform of duration $T_c = \frac{T}{N}$ and with unit energy [i.e., $\int_0^{T_c} \psi(t)^2 dt = 1$]. At the receiver, the received signal r(t) is filtered by a chip-matched filter and then sampled at the chip rate. The sample corresponding to the j^{th} chip of the i^{th} symbol is given by

$$r_j[i] \triangleq \int_{iT+jT_c}^{iT+(j+1)T_c} r(t)\psi(t - iT - jT_c)dt, j = 0, \dots, N-1;$$
$$i = 0, \dots, M-1 \quad (3)$$

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The resulting discrete-time signal corresponding to the *i*th symbol is then given by

$$r[i] = \sum_{k=1}^{K} A_k b_k[i] s_k + n[i] = SAb[i] + n[i] \tag{4}$$

Suppose that we are interested in demodulating the data bits of a particular user, say user $\{b_k[i]\}_{i=0}^{M-1}$, based on the received waveforms $\{r[i]\}_{i=0}^{M-1}$. A linear receiver for this purpose is described by a weight vector, such that the desired user's data bits are demodulated according to

$$z_1[i] = w_1^H r[i] \tag{5}$$

$$\hat{b}_1[i] = sign\{\Re(A_1^* z_1[i])\} \tag{6}$$

In case that the complex amplitude A_1 of the desired user is unknown, we can resort to differential detection. Define the differential bit as

$$\beta_1[i] \triangleq b_1[i] b_1[i-1] \tag{7}$$

Then using the linear detector output, the following differential detection rule can be used

$$\hat{\beta}_1 = sign\{\Re(z_1[i] z_1[i-1]^*)\} \tag{8}$$

Substituting **r[**i] in z_1 [i]the output of the linear receiver $\mathbf{w}_1$ can be written as

$$z_1[i] = A_1(\boldsymbol{w}_1^H \boldsymbol{s}_1) b_1[i] + \sum_{k=2}^{K} A_k(\boldsymbol{w}_1^H \boldsymbol{s}_k) b_k[i] + \boldsymbol{w}_1^H \boldsymbol{n}[i] \tag{9}$$

The first term contains the useful signal of the desired user; the second term contains the signals from other undesired users the so-called multiple-access interference (MAI); and the last term contains the ambient Gaussian noise. The simplest linear receiver is the conventional matched-filter, where $w_1 = s_1$. A matched filter receiver is optimal only in a single-user channel (i.e. *K*=1). In a multiuser channel (i.e., K> 1), this receiver may perform poorly since it makes no attempt to ameliorate the MAI, a limiting source of interference in multiple-access channels.

III. Bling And Group Blind Detectors

A. Blind multiuser detectors

Consider the signal model (8). The linear MMSE detector for user 1 is defined as

$$\boldsymbol{w}_1 = \min_{\boldsymbol{w} \in \mathbb{R}^N} E\left\{\left(\frac{1}{\mu} b_1[i] - \boldsymbol{w}^T \boldsymbol{r}[i]\right)^2\right\} = \frac{A_1}{\mu} \boldsymbol{C}_r^{-1} \boldsymbol{s}_1 \tag{10}$$

with

$$\boldsymbol{C}_r \triangleq E\{\boldsymbol{r}[i]\boldsymbol{r}[i]^H\} = \boldsymbol{S}|\boldsymbol{A}|^2 \boldsymbol{S}^H + \sigma^2 \boldsymbol{I}_N \tag{11}$$

where μ is some positive constant. The linear detection rule given by

$$\hat{b}_1[i] = sign(\boldsymbol{w}_1^T \boldsymbol{r}[i]) \tag{12}$$

is invariant to a positive scaling, the linear detector in (10) is invariant to the positive constant . For simplicity, we choose $\mu = A_1$ so that $\boldsymbol{w}_1 = \boldsymbol{C}_r^{-1} s_1$. Let the eigen decomposition [4] of $\mathbf{C}_r$ in (11) be

$$\boldsymbol{C}_r = \boldsymbol{U}_s \boldsymbol{\Lambda}_s \boldsymbol{U}_s^T + \sigma^2 \boldsymbol{U}_n \boldsymbol{U}_n^T \tag{13}$$

Where $\boldsymbol{\Lambda}_s = diag(\lambda_1, \cdots, \lambda_K)$ contains the largest K eigen values of, $\boldsymbol{C}_r$, $\boldsymbol{U}_s = [\boldsymbol{u}_1, \cdots, \boldsymbol{u}_K]$ contains the K orthogonal eigenvectors corresponding to the largest K Eigen values in $\boldsymbol{\Lambda}_s$; $\boldsymbol{U}_n = [\boldsymbol{u}_{K+1}, \cdots, \boldsymbol{u}_N]$ contains the (n-k) orthogonal Eigen vectors corresponding to the smallest Eigen σ^2 value of $\mathbf{c}_r$. It is easy to see that range(**S**) = range ($\mathbf{U}_s$) . The column space of $\mathbf{U}_s$ is called the signal subspace and its orthogonal complement, the noise subspace, is spanned by the columns of $\mathbf{U}_n$. Corresponding to two forms of the linear MMSE (10) and (12), there are two approaches to its blind implementation i.e., the implementation which assumes knowledge of only the signature waveform $\mathbf{s}_1$ of the desired user.

DMI blind linear MMSE detector

In this method [3], [1], the autocorrelation matrix $\boldsymbol{C}_r$ in (10) is replaced by the corresponding sample estimate.

Compute the detector

$$\hat{\boldsymbol{C}}_r = \frac{1}{M} \sum_{i=0}^{M-1} \boldsymbol{r}[i] \boldsymbol{r}[i]^H \tag{14}$$

$$\hat{\boldsymbol{m}}_1 = \hat{\boldsymbol{C}}_r^{-1} \boldsymbol{s}_1 \tag{15}$$

Perform the differential detection

$$z_1[i] = \hat{\boldsymbol{m}}_1^H \boldsymbol{r}[i] \tag{16}$$

$$\hat{\beta}_1[i] = sign\{\Re(z_1[i] z_1[i-1]^*)\}, \quad i = 1, \ldots M-1 \tag{17}$$

The above algorithm is a batch processing method, i.e., it computes the detector only once based on a block of received signals $\{\boldsymbol{r}[i]\}_{i=0}^{M-1}$; and the estimated detector is then used to detect all data bits of the desired user contained in the same signal block, $\{b_1[i]\}_{i=0}^{M-1}$. The idea is to perform sequential detector estimation and data detection. That is, suppose that at time $(i-1)$, an estimated detector $\boldsymbol{m}_1[i-1]$ is employed to detect the data bit $b_1[i-1]$. At time i, a new signal $\boldsymbol{r}[i]$ is received which is then used to update the detector estimate to obtain $\boldsymbol{m}_1[i]$. The updated detector is used to detect the data bit $b_1[i]$. Hence the blind detector is sequentially updated at the symbol rate.

Subspace blind linear MMSE detector

The linear MMSE detector $\boldsymbol{w}_1$ in (10) can also be written in terms of the signal subspace components as [4], [5]

$$\boldsymbol{w}_1 = \boldsymbol{U}_s \boldsymbol{\Lambda}_s^{-1} \boldsymbol{U}_s^T \boldsymbol{s}_1 \tag{18}$$

Now the eigencomponent $\boldsymbol{\Lambda}_s$ and $\boldsymbol{U}_s$ are replaced by the corresponding eigenvalues and eigenvectors of the sample autocorrelation matrix $\hat{\boldsymbol{C}}_r$.

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- Compute the detector

$$\widehat{C}_r = \frac{1}{M}\sum_{i=0}^{M-1} r[i]r[i]^H = \widehat{U}_s\widehat{\Lambda}_s\widehat{U}_s^H + \widehat{U}_n\widehat{\Lambda}_n\widehat{U}_n^H \quad (19)$$

$$\widehat{w}_1 = U_s\Lambda_s^{-1}U_s^T s_1 \quad (20)$$

- Perform differential detection

$$z_1[i] = \widehat{w}_1^H r[i] \quad (21)$$

$$\hat{\beta}_1[i] = sign\{\Re(z_1[i]z_1[i-1]^*)\}, \quad i = 1, \dots M-1 \quad (22)$$

B. Group blind multiuser detectors

In group-blind multiuser detection, it is assumed that the receiver has the knowledge of the signature waveforms of some but not all the users. Without loss of generality, assume that the first K users' signature waveforms are known to the receiver, whereas those of the rest $(K - \bar{K})$ users' are unknown. Denote $\bar{S} \triangleq [s_1 \cdots s_{\bar{k}}]$. It is assumed that $\bar{S}$ has full column rank. Denot $\bar{e}_k$ as the k^{th} unit vector in $\mathbb{R}^{\bar{K}}$. The group-blind linear hybrid detector zero-forces the interference caused by the $\bar{K}$ known users' and suppresses that from the rest $(K - \bar{K})$ unknown users according to the MMSE criterion. In particular, such a detector for user 1 is given by the solution to the following constrained optimization problem

$$w_1 = \arg\min_{w\in\mathbb{R}^N} E\{(A_1 b_1[i] - w^T r[i])^2\}, \ s.t\ w^T\bar{S} = \tilde{e}_1^T \quad (23)$$

The solution to (23) in terms of the signal subspace components of C_r is

$$w_1 = U_s\Lambda_s^{-1}U_s^H\tilde{S}\left(\tilde{S}^H U_s\Lambda_s^{-1}U_s^H\tilde{S}\right)^{-1}\tilde{e}_1 \quad (24)$$

E.Group blind linear MMSE detector

This detector is formed by replacing the eigen components

$$\widehat{C}_r = \frac{1}{M}\sum_{i=0}^{M-1} r[i]r[i]^H = \widehat{U}_s\widehat{\Lambda}_s U_S^H + \widehat{U}_n\widehat{\Lambda}_n\widehat{U}_n^{\ H} \quad (25)$$

of C_r in (24) by those of the corresponding sample correlation matrix $\widehat{C}_r$.

- Compute the detector

$$w_1 = \widehat{U}_s\widehat{\Lambda}_s^{-1}\widehat{U}_s^T\tilde{S}\left(\tilde{S}^H\widehat{U}_s\widehat{\Lambda}_s^{-1}\widehat{U}_s^H\tilde{S}\right)^{-1}\tilde{e}_1 \quad (26)$$

- Perform the differential detection

$$\hat{\beta}_1[i] = sign\{\Re(z_1[i]z_1[i-1]^*)\}, \quad i = 1, \dots M-1 \quad (27)$$

$$z_1[i] = \widehat{w}_1^H r[i] \quad (28)$$

IV. Performance Measures

Consider the signal model (4). Here we consider user 1 is the user of interest. A linear detector for user 1 is a (deterministic) vector $w \in \mathbb{R}^N$ such that $b_1[i]$ is demodulated according to (12). Now considering that the user bit streams are independent in (9) the SINR at the output of the linear detector w_1 is given by

$$SINR(w_1) = \frac{E\{w_1^T r[i]|b_1[i]\}^2}{E\{Var\{w_1^T r[i]|b_1[i]\}}$$

$$= \frac{A_1^2(w_1^T s_1)^2}{\sum_{k=2}^K A_k^2(w_1^T s_k)^2 + \sigma^2\|w_1\|^2} \quad (29)$$

Now suppose that an estimate w_1 of the weight vector w_1 is obtained from the received signals $\{r[i]\}_{i=0}^{M-1}$. Denote $\Delta w_1 \triangleq \widehat{w}_1 - w_1$. Obviously both $\widehat{w}_1$ and Δw_1 are random vectors and are functions of the random quantities $\{b[i], n[i]\}_{i=0}^{M-1}$. In typical adaptive multiuser detection scenarios, the estimated detector $\widehat{w}_1$ is employed to demodulate future received signals, say $r[j],\ j > M$. Then the output is given by

$$\widehat{w}_1^T r[j] = w_1^T r[j] + \Delta w_1^T r[j],\ j > M, \quad (30)$$

Where the first term represents the output of the true weight vector w_1. The second term represents an additional noise term caused by the estimation error Δw_1. Hence from the above equation, the average SINR [10] at the output of any unbiased estimated linear detector $\widehat{w}_1$ [11], [12] is given by

$$\overline{SINR(\widehat{w}_1)} = \frac{A_1^2(w_1^T s_1)^2}{\sum_{k=2}^K A_k^2(w_1^T s_k)^2 + \sigma^2\|w_1\|^2 + E\{(\Delta w_1^T r[j])^2\}} \quad (31)$$

With

$$E\{(\Delta w_1^T r[j])^2\} = tr(E\{\Delta w_1^T r[j] r[j]^T \Delta w_1\})$$
$$= \frac{1}{M} tr(C_w C_r) \quad (32)$$

where $\frac{1}{M}C_w \triangleq E\{\Delta w_1 \Delta w_1^T\}$ and $C_r \triangleq E\{r[j]r[j]^T\}$. Since Δw_1 is function of $\{r[i]\}_{i=1}^M$, for fixed i, Δw_1 and r[i] are in general, correlated. For large M such a correlation is small. Therefore we can still use (31) and (32) as approximate SINR expression.

For the group blind detectors as the known users interference [13], [11], [12] is suppressed to zero the SINR expression slightly varies and is as follows

$$\overline{SINR(\widehat{w}_1)} = \frac{A_1^2}{\sum_{k=2}^{K-\bar{K}} A_{\bar{K}+k}^2\left(w_1^T s_{\bar{K}+k}\right)^2 + \sigma^2\|w_1\|^2 + \mathrm{tr}(C_w C_r)} \quad (33)$$

V. Simulation Results

Here using (31), (32) and (33) and substituting into corresponding weight vectors for DMI , Subspace and Group blind hybrid detectors[14].The SINR is calculated for equicorrelate d signals with power control for mathematical simplicity.

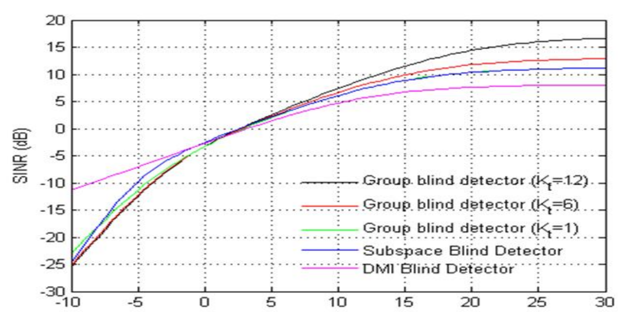

Fig 1 Average output SINR versus SNR for a hybrid group blind detector and two blind detectors. N = 32, M = 200, K=16, ρ = 0.4. (In the figure $K_t \triangleq \bar{K}$.)

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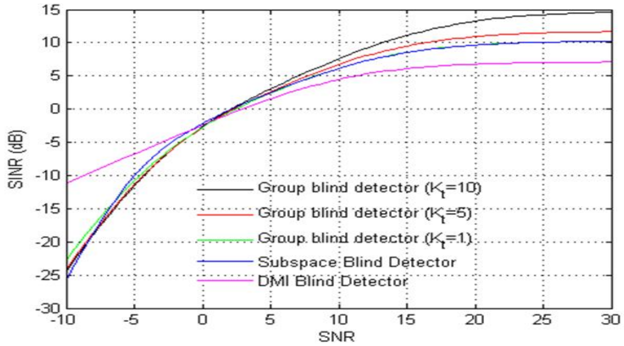

Fig 2 Average output SINR versus SNR for a hybrid group blind detector and two blind detectors. N = 30, M = 150, K=15, ρ = 0.3. (In the figure $K_t \triangleq \bar{K}$.)

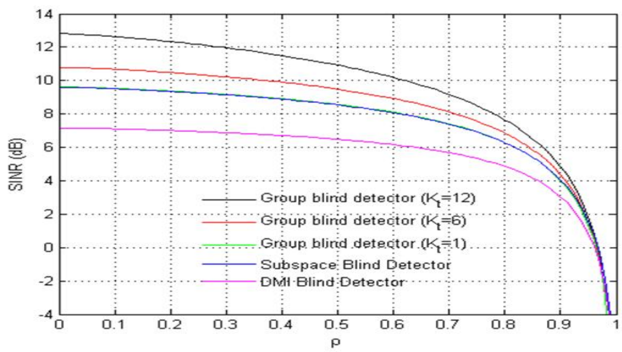

Fig 3 Average output SINR versus ρ for a hybrid group blind detector and two blind detectors. N = 32, M = 200, K=16, SNR = 15dB. (In the figure $K_t \triangleq \bar{K}$.)

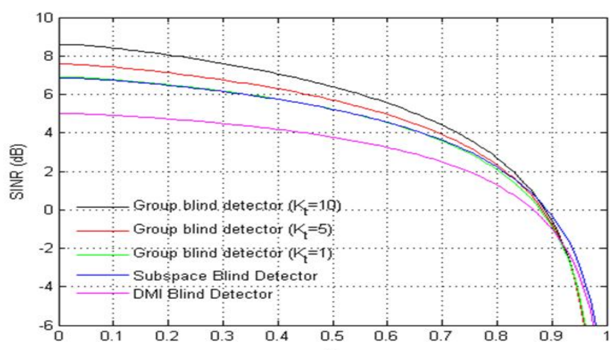

Fig 4 Average output SINR versus ρ for a hybrid group blind detector and two blind detectors. N = 30, M = 150, K=15, SNR = 10dB. (In the figure $K_t \triangleq \bar{K}$.)

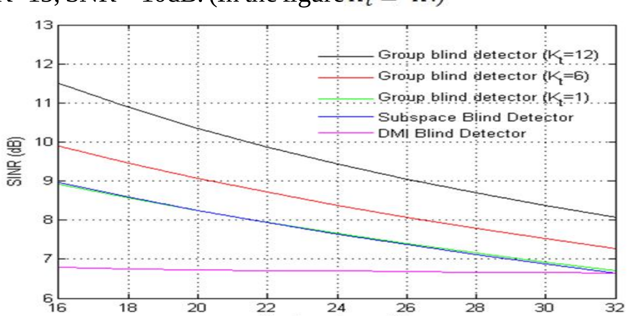

Fig 5 Average output SINR versus K for a hybrid group blind detector and two blind detectors. N = 32, M = 200, SNR = 15 dB, ρ = 0.4. (In the figure $K_t \triangleq \bar{K}$.)

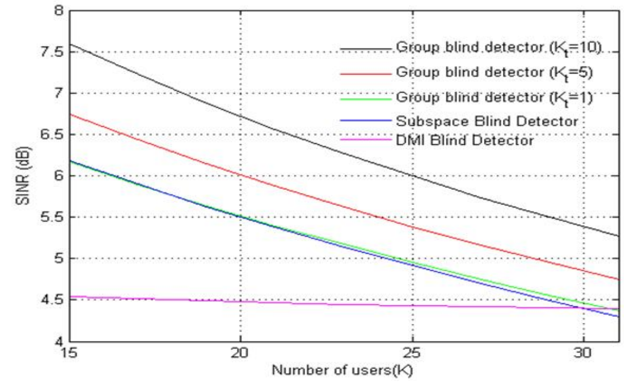

Fig 6 Average output SINR versus K for a hybrid group blind detector and two blind detectors. N = 30, M = 150, SNR = 10 dB, ρ = 0.3. (In the figure $K_t \triangleq \bar{K}$.)

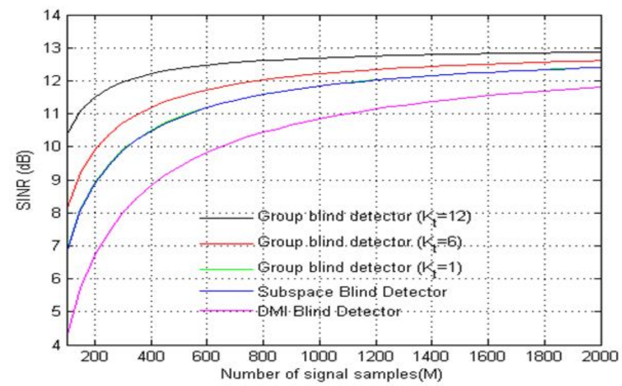

Fig 7 Average output SINR versus number of samples (M) for a hybrid group blind detector and two blind detectors. N = 32, SNR = 15 dB, K=16, ρ = 0.4. (In the figure $K_t \triangleq \bar{K}$.)

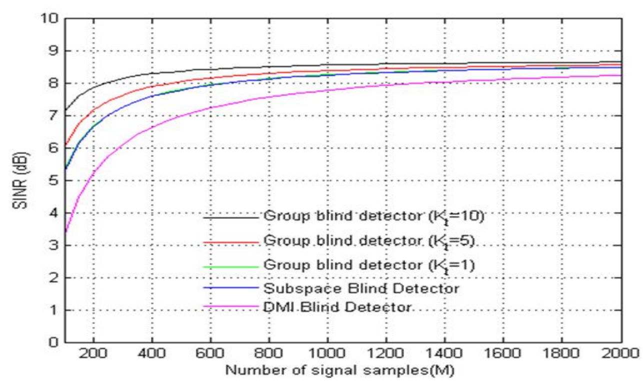

Fig. 8. Average output SINR versus number of samples (M) for a hybrid group blind detector and two blind detectors. N = 30, SNR = 10 dB, K=15, ρ = 0.3. (In the figure $K_t \triangleq \bar{K}$.)

VI. Conclusion

In this paper, the performance of blind multiuser detectors is analyzed. The blind multiuser detectors considered are DMI, subspace and group blind detector. The analysis is performed by means of the Signal to Interference Noise Ratio(SINR) and Bit Error Rate(BER) obtained during our simulation. We have also compared the performance of three different blind detectors

From figures 1 and 2 shows the SINR increases with

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increase in SNR, the SINR of group blind detector and subspace detectors is less than that of DMI detector in negative SNR region while SNR increases the group blind detector outperform the DMI detector.

Figures 3 and 4 observed that the SINR of group blind detector is higher than that of DMI and Subspace detectors.

The SINR of all the detectors deteriorates with increase in ρ (signal cross correlation).

From figures 5 and 6 it is seen that SINR of all detectors reduces with increase in number of users(K).Particularly in DMI detector ,the SINR decreases slowly than the other detectors. In less user system the group blind detector has outperformed the DMI and subspace detectors.

Figures 7 and 8 depict the SINR of all detectors gets better with increase in number of samples. The group blind multiuser detector SINR is better than the others.

In general it is observed from figures 1, 2, 3, 4, 5, 6, 7, 8 the SINR of group blind detector has increased with increase in known users.

In all cases the DMI blind detector is the poor performing detector than the sub space and group blind detector. So it can be concluded that the group blind detector offer significant performance gains where more number of users are known. So they are used in uplink environment typically a base station.

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